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Probabilistic water reservoir operation using implicit stochastic optimization and vine copula functions

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Introduction

Decisions in reservoir operation problems deal with the amount of water that should be released and stored over a period of time considering the variation and uncertainties of future streamflows and demands (Nagesh Kumar and Janga Reddy 2007)

Optimization techniques used in reservoir systems can be classified into two types. Explicit stochastic optimization (ESO) and implicit stochastic optimization (ISO).

ESO considers the uncertainties of streamflows and other parameters of the problems in an explicit way. This approach is commonly used when inflows cannot be reliably forecasted for a relatively long period, requiring the use of probability distribution functions to represent uncertainties of the data.

ISO derives operational policies of reservoirs based on deterministic models (Zambelli et al. 2006). This approach takes into consideration the use of independent inflow scenarios, providing an optimal solution for each one.

Copulas are employed to relate hydrological variables that affect the operation of water reservoirs and used to derive probabilistic long-term operational policies for a single hydropower reservoir located in a semiarid region of Brazil.

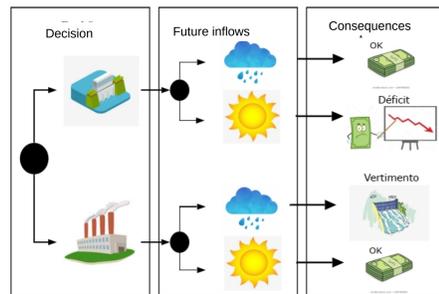


Fig. 1 General framework of a hydrothermal power system

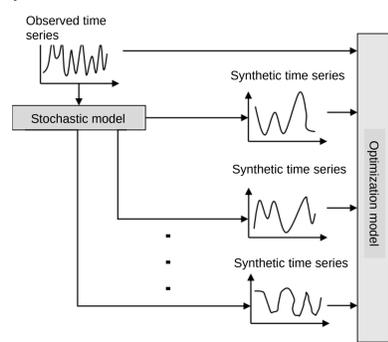


Fig. 2 Stochastic optimization procedure for hydrothermal operation

Methodology

This study was conducted into three stages:

- 1) Simulate monthly streamflow scenarios based on a periodic vine copula-entropy model
- 2) Compute optimal releases policies using an ISO approach
- 3) Estimate reservoir operational policies based on a probabilistic simulation process with copulas

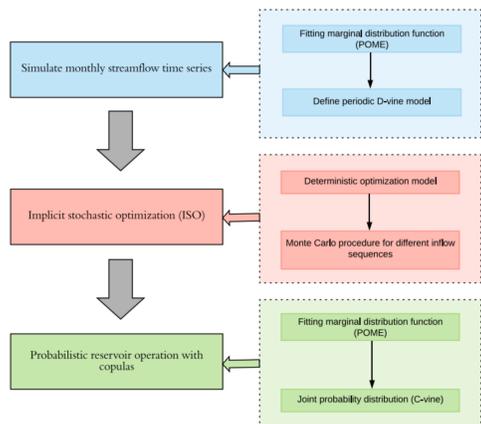


Fig. 3 General framework to derive reservoir operation policies combining ISO and copulas

Copula theory

A copula C is a multivariate distribution function with marginals as the uniformly distributed U(0, 1) (Joe 1996)

$$H(x_1, \dots, x_d) = C[F_1(x_1), \dots, F_d(x_d)] = C(u_1, \dots, u_d)$$

For dimensions greater than two, vines copulas are commonly organized by a set of trees composed by edges and vertices. Two special vines (C-vine and D-vine)

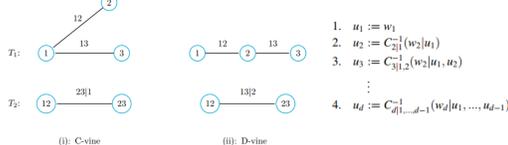


Fig. 4 Three-dimensional vine copula construction

Principle of Maximum Entropy

For a random variable X, the most probable probability density function (PDF) is the one that maximizes the Shannon entropy H(x) defined as:

$$H(x) = - \int_a^b f(x) \log f(x) dx$$

$$\int_a^b f(x) dx = 1 \quad f(x) \geq 0 \quad \forall x \in [a, b]$$

$$\int_a^b h_i f(x) dx = \bar{h}_i \quad i = 1, \dots, m$$

According to Kapur and Kesavan (1992), the maximum entropy-based (ME-based) PDF of X can be obtained as follows:

$$f(x) = \exp \left[- \ln \left(\int_a^b \exp \left(- \sum_{i=1}^m \lambda_i h_i(x) \right) dx \right) - \sum_{i=1}^m \lambda_i h_i(x) \right]$$

Optimization model

The main objective of the operation is to find the allocations of water that best satisfy their respective demands without compromising the systems.

$$\text{minimize } \sum_{i=1}^N \left[\frac{R(t) - D(t)}{D(t)} \right]^2$$

subject to

$$S(t) = S(t-1) + I(t) - E(t) - R(t) - S_p(t); \quad \forall t$$

$$0 \leq R(t) \leq D(t); \quad \forall t$$

$$S_{min} \leq S(t) \leq S_{max}; \quad \forall t$$

$$S_p(t) \geq 0; \quad \forall t$$

Study case

The Sobradinho reservoir is located in the Northeastern region of Brazil, has a surface area of 4.214 km² and a storage capacity of 34.1 km³ approximately.

This reservoir encloses the waters of the Sao Francisco River, which is the longest river that runs entirely in Brazilian territory, with a mainstream length of 2.830 km and a drainage area of 641.000 km².

The Sobradinho reservoir has dead and a maximum storage volume equal to 5.447 hm³ and 34.116 hm³ respectively.

Monthly streamflow records from 1931 to 2017 at the Sobradinho hydropower station were used in this study. The streamflow data was provided by the Brazilian National Electrical System Operator (ONS)

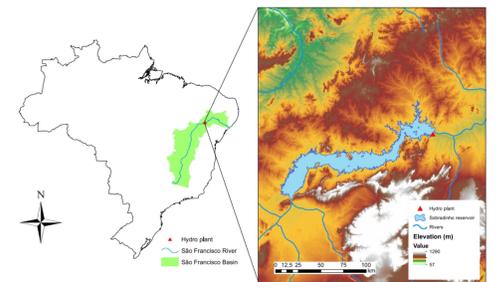


Fig. 5 Location of the Sobradinho reservoir

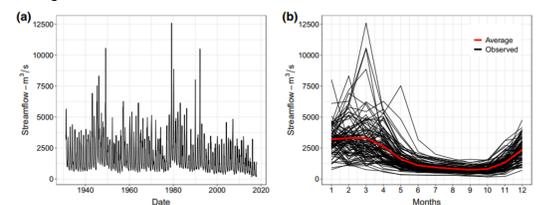


Fig. 6 (a) Monthly streamflow time series and (b) annual cycle in Sao Francisco River

Results and discussions

Figure 6 presents 300 simulated scenarios (grey lines), each one containing 60 months, generated by the periodic vine-copula entropy-based model in Sao Francisco river

A Monte Carlo process was executed over an operating horizon of 1320 months (110 years) for 70 inflow sequences.

The initial storage was set to S_{max}. The monthly demand D(t) of the objective function was assumed to be the reservoir yield at 95% reliability

Initial storage, inflow, and water release values were grouped month by month to construct the respective operational curves.

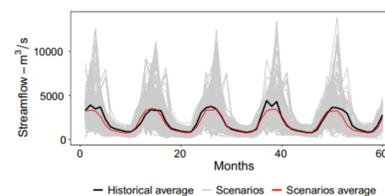


Fig. 7 Simulated streamflow scenarios in Sao Francisco river

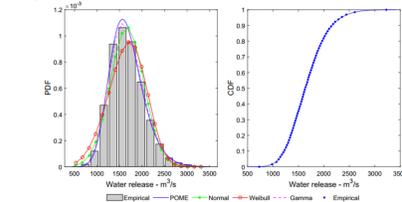


Fig. 8 Comparison of theoretical and empirical marginal density function (PDF) and cumulative distribution function (CDF) of water

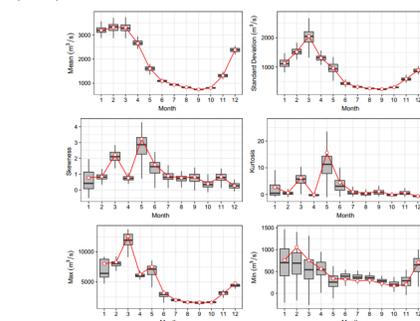


Fig. 9 Comparison of monthly statistics of simulated (boxplot) and observed (red line) streamflow data in Sao Francisco river

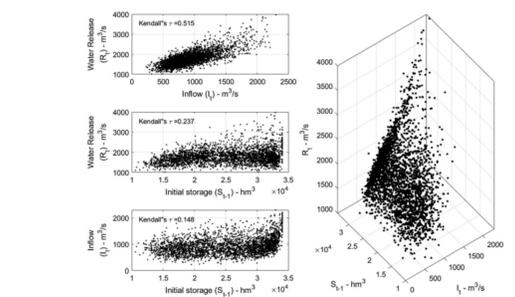


Fig. 10 Scatterplots of optimized operating reservoir variables in June

A set of joint probability distribution functions is constructed using a vine copula approach. For each variable, a marginal ME-based distribution function is estimated using the first four statistical moments as constraints.

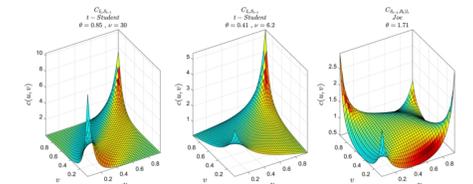


Fig. 11 Copula density surfaces associated to the C-vine structure of June

A simulation procedure based on copula was performed to forecast the expected amount of water that should be released in the Sobradinho reservoir one-month ahead. For each step, the performed model assumes the prior knowledge of the initial storage volume and the future inflow conditions in the river.

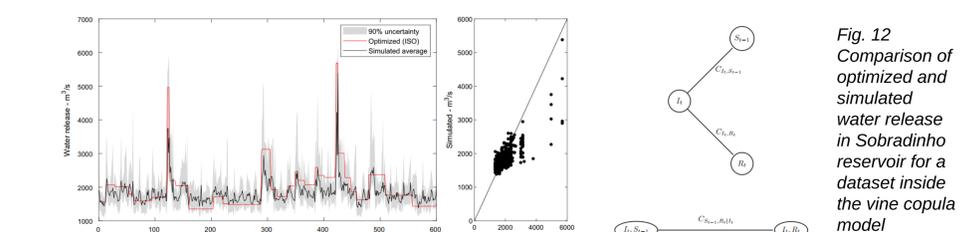


Fig. 12 Comparison of optimized and simulated water release in Sobradinho reservoir for a dataset inside the vine copula model

Considering the optimized data as observed values, the relative error between simulated and optimized values is 11%, the calculated NSE is 0.55 and the RMSE is 350 m³/s approximately. The main observed disadvantage of the proposed model is the randomness presented by simulated values, increasing the variability of the results when it is compared with optimized data.

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Acknowledgements

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